```
> v := array(1..8) :for i to 8 do v[i] := i^2 od: print(v);
```

$$\left[\begin{array}{cccccccc} 1 & 4 & 9 & 16 & 25 & 36 & 49 & 64 \end{array}\right] \quad (1)$$

```
> A := array(1..2, 1..2) : A[1, 1] := cos(x) : A[1, 2] := sin(x) : A[2, 1] := -A[1, 2] : A[2,
    2] := A[1, 1] : print(A);
```

$$\left[\begin{array}{cc} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{array}\right] \quad (2)$$

```
> map(diff, A, x);
```

$$\left[\begin{array}{cc} -\sin(x) & \cos(x) \\ -\cos(x) & -\sin(x) \end{array}\right] \quad (3)$$

```
> # Multiplication sans le package Linear Algebra (à ne pas retenir)
> A := matrix(3, 3, [5, -1, 0, 0, 1, 9, 4, 6, 3]);
```

$$A := \left[\begin{array}{ccc} 5 & -1 & 0 \\ 0 & 1 & 9 \\ 4 & 6 & 3 \end{array}\right] \quad (4)$$

```
> B := matrix(3, 3, [[9, 3, -6], [3, 2, 0], [9, 8, 7]]);
```

$$B := \left[\begin{array}{ccc} 9 & 3 & -6 \\ 3 & 2 & 0 \\ 9 & 8 & 7 \end{array}\right] \quad (5)$$

```
> evalm(A&*B);
```

$$\left[\begin{array}{ccc} 42 & 13 & -30 \\ 84 & 74 & 63 \\ 81 & 48 & -3 \end{array}\right] \quad (6)$$

```
>  evalm(A&*B - B&*A);
```

$$\left[\begin{array}{ccc} 21 & 55 & -39 \\ 69 & 75 & 45 \\ 8 & 7 & -96 \end{array}\right] \quad (7)$$

```
> inverse(A)
```

$$\left[\begin{array}{ccc} \frac{17}{97} & -\frac{1}{97} & \frac{3}{97} \\ -\frac{12}{97} & -\frac{5}{97} & \frac{15}{97} \\ \frac{4}{291} & \frac{34}{291} & -\frac{5}{291} \end{array}\right] \quad (8)$$

```
> evalm(A&*inverse(A))
```

(9)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (9)$$

```
> restart
> # Utilisation de LinearAlgebra -> à retenir
> with(LinearAlgebra)
```

[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, (10)
BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column,
ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix,
CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy,
CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal,
DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues,
Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm,
FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic,
GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix,
HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix,
IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary,
JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition,
LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential,
MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower,
MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply,
NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot,
PopovForm, ProjectionMatrix, QRDecomposition, RandomMatrix, RandomVector, Rank,
RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation,
RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues,
SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis,
SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm,
UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply,
VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip]

```
> A := Matrix(3, 3, [5, -1, 0, 0, 1, 9, 4, 6, 3]);
```

$$A := \begin{bmatrix} 5 & -1 & 0 \\ 0 & 1 & 9 \\ 4 & 6 & 3 \end{bmatrix} \quad (11)$$

```
> B := Matrix(3, 3, [[9, 3, -6], [3, 2, 0], [9, 8, 7]]);
```

$$B := \begin{bmatrix} 9 & 3 & -6 \\ 3 & 2 & 0 \\ 9 & 8 & 7 \end{bmatrix} \quad (12)$$

> $A \cdot B$

$$\begin{bmatrix} 42 & 13 & -30 \\ 84 & 74 & 63 \\ 81 & 48 & -3 \end{bmatrix} \quad (13)$$

> $A \cdot B - B \cdot A$

$$\begin{bmatrix} 21 & 55 & -39 \\ 69 & 75 & 45 \\ 8 & 7 & -96 \end{bmatrix} \quad (14)$$

> $A^5 + B^5$

$$\begin{bmatrix} -196441 & -94936 & 64056 \\ -12939 & -11892 & 28791 \\ -54590 & -96073 & -143330 \end{bmatrix} \quad (15)$$

> $Id := IdentityMatrix(3)$

$$Id := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (16)$$

> $restart; with(LinearAlgebra)$

[*&x*, *Add*, *Adjoint*, *BackwardSubstitute*, *BandMatrix*, *Basis*, *BezoutMatrix*, *BidiagonalForm*, *BilinearForm*, *CARE*, *CharacteristicMatrix*, *CharacteristicPolynomial*, *Column*, *ColumnDimension*, *ColumnOperation*, *ColumnSpace*, *CompanionMatrix*, *CompressedSparseForm*, *ConditionNumber*, *ConstantMatrix*, *ConstantVector*, *Copy*, *CreatePermutation*, *CrossProduct*, *DARE*, *DeleteColumn*, *DeleteRow*, *Determinant*, *Diagonal*, *DiagonalMatrix*, *Dimension*, *Dimensions*, *DotProduct*, *EigenConditionNumbers*, *Eigenvalues*, *Eigenvectors*, *Equal*, *ForwardSubstitute*, *FrobeniusForm*, *FromCompressedSparseForm*, *FromSplitForm*, *GaussianElimination*, *GenerateEquations*, *GenerateMatrix*, *Generic*, *GetResultDataType*, *GetResultShape*, *GivensRotationMatrix*, *GramSchmidt*, *HankelMatrix*, *HermiteForm*, *HermitianTranspose*, *HessenbergForm*, *HilbertMatrix*, *HouseholderMatrix*, *IdentityMatrix*, *IntersectionBasis*, *IsDefinite*, *IsOrthogonal*, *IsSimilar*, *IsUnitary*, *JordanBlockMatrix*, *JordanForm*, *KroneckerProduct*, *LA_Main*, *LUDecomposition*, *LeastSquares*, *LinearSolve*, *LyapunovSolve*, *Map*, *Map2*, *MatrixAdd*, *MatrixExponential*, *MatrixFunction*, *MatrixInverse*, *MatrixMatrixMultiply*, *MatrixNorm*, *MatrixPower*, *MatrixScalarMultiply*, *MatrixVectorMultiply*, *MinimalPolynomial*, *Minor*, *Modular*, *Multiply*, *NoUserValue*, *Norm*, *Normalize*, *NullSpace*, *OuterProductMatrix*, *Permanent*, *Pivot*, *PopovForm*, *ProjectionMatrix*, *QRDecomposition*, *RandomMatrix*, *RandomVector*, *Rank*, *RationalCanonicalForm*, *ReducedRowEchelonForm*, *Row*, *RowDimension*, *RowOperation*, *RowSpace*, *ScalarMatrix*, *ScalarMultiply*, *ScalarVector*, *SchurForm*, *SingularValues*, (17)

SmithForm, *SplitForm*, *StronglyConnectedBlocks*, *SubMatrix*, *SubVector*, *SumBasis*, *SylvesterMatrix*, *SylvesterSolve*, *ToeplitzMatrix*, *Trace*, *Transpose*, *TridiagonalForm*, *UnitVector*, *VandermondeMatrix*, *VectorAdd*, *VectorAngle*, *VectorMatrixMultiply*, *VectorNorm*, *VectorScalarMultiply*, *ZeroMatrix*, *ZeroVector*, *Zip*]

> $M := Matrix([[1, -1, 1, 7], [-7, -6, 0, 0], [0, 1, 1, -4], [2, 1, 6, 3]]); B := Matrix([[0, 0], [0, 1], [1, 0], [-1, -1]]); C := Matrix([[1, 2, 2, 1], [4, 5, 4, 5]])$

$$M := \begin{bmatrix} 1 & -1 & 1 & 7 \\ -7 & -6 & 0 & 0 \\ 0 & 1 & 1 & -4 \\ 2 & 1 & 6 & 3 \end{bmatrix}$$

$$B := \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ -1 & -1 \end{bmatrix}$$

$$C := \begin{bmatrix} 1 & 2 & 2 & 1 \\ 4 & 5 & 4 & 5 \end{bmatrix} \quad \textbf{(18)}$$

> $M \cdot B; C \cdot M; B \cdot C; C \cdot B; M + B \cdot C$

$$\begin{bmatrix} -6 & -8 \\ 0 & -6 \\ 5 & 5 \\ 3 & -2 \end{bmatrix}$$

$$\begin{bmatrix} -11 & -10 & 9 & 2 \\ -21 & -25 & 38 & 27 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 4 & 5 & 4 & 5 \\ 1 & 2 & 2 & 1 \\ -5 & -7 & -6 & -6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 7 \\ -3 & -1 & 4 & 5 \\ 1 & 3 & 3 & -3 \\ -3 & -6 & 0 & -3 \end{bmatrix} \quad \textbf{(19)}$$

> $MatrixInverse(M)$

$$\begin{bmatrix} \frac{162}{23} & \frac{1}{23} & \frac{234}{23} & -\frac{66}{23} \\ -\frac{189}{23} & -\frac{5}{23} & -\frac{273}{23} & \frac{77}{23} \\ \frac{1}{23} & \frac{1}{23} & \frac{4}{23} & \frac{3}{23} \\ -\frac{47}{23} & -\frac{1}{23} & -\frac{73}{23} & \frac{20}{23} \end{bmatrix} \quad (20)$$

> $Determinant(M)$

$$-23 \quad (21)$$

> $A := Matrix([[1, 2, 3, 4, 5], [6, 7, 7, 7, 6], [1, 1, 1, 1, 1], [3, 2, 1, 2, 3], [1, -1, -1, -1, -1]])$

$$A := \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 7 & 7 & 6 \\ 1 & 1 & 1 & 1 & 1 \\ 3 & 2 & 1 & 2 & 3 \\ 1 & -1 & -1 & -1 & -1 \end{bmatrix} \quad (22)$$

> $Determinant(A)$

$$4 \quad (23)$$

> $MatrixInverse(A)$

$$\begin{bmatrix} 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ -\frac{1}{2} & 0 & 0 & \frac{1}{2} & -1 \\ 0 & -1 & 9 & -1 & 0 \\ \frac{1}{2} & 2 & -15 & \frac{1}{2} & 1 \\ 0 & -1 & \frac{13}{2} & 0 & -\frac{1}{2} \end{bmatrix} \quad (24)$$

> $V := VandermondeMatrix([t, u, v, w])$

$$V := \begin{bmatrix} 1 & t & t^2 & t^3 \\ 1 & u & u^2 & u^3 \\ 1 & v & v^2 & v^3 \\ 1 & w & w^2 & w^3 \end{bmatrix} \quad (25)$$

> $p := Determinant(V)$

$$p := t^3 u^2 v - t^3 u^2 w - t^3 u v^2 + t^3 u w^2 + t^3 v^2 w - t^3 v w^2 - t^2 u^3 v + t^2 u^3 w + t^2 u v^3 - t^2 u w^3 \quad (26)$$

$$-t^2 v^3 w + t^2 v w^3 + t u^3 v^2 - t u^3 w^2 - t u^2 v^3 + t u^2 w^3 + t v^3 w^2 - t v^2 w^3 - u^3 v^2 w + u^3 v w^2 + u^2 v^3 w - u^2 v w^3 - u v^3 w^2 + u v^2 w^3$$

> $factor(p)$

$$(v-w)\,(u-w)\,(u-v)\,(t-w)\,(t-v)\,(t-u) \tag{27}$$

> $A := Matrix(3, 4, [0, 2, 1, -2, 3, 5, -5, 1, 2, 4, -2, 2]);\ b := Vector([-2, 1, 2]);$
$LinearSolve(A, b);$

$$A := \begin{bmatrix} 0 & 2 & 1 & -2 \\ 3 & 5 & -5 & 1 \\ 2 & 4 & -2 & 2 \end{bmatrix}$$

$$b := \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 7 - 7\,\_t3_4 \\ -2 + 2\,\_t3_4 \\ 2 - 2\,\_t3_4 \\ \_t3_4 \end{bmatrix} \tag{28}$$

> $eqns := \{x + y + 2\cdot z = 9, 3\cdot x - 2\cdot y = 6, 4\cdot x + 7\cdot y = 0\};$

$$eqns := \{3x - 2y = 6, 4x + 7y = 0, x + y + 2z = 9\} \tag{29}$$

> $A, b := GenerateMatrix(eqns, [x, y, z]);$

$$A, b := \begin{bmatrix} 3 & -2 & 0 \\ 4 & 7 & 0 \\ 1 & 1 & 2 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ 9 \end{bmatrix} \tag{30}$$

> $GenerateEquations(A, [x, y, z], b)$

$$[3x - 2y = 6, 4x + 7y = 0, x + y + 2z = 9] \tag{31}$$

> $eqns := \{2\,x1 - 4\,x2 + 3\,x3 - 4\,x4 - 11\,x5 = 28,\ -x1 + 2\,x2 - x3 + 2\,x4 + 5\,x5 = -1,$
$-3\,x3 + x4 + 6\,x5 = -10, 3\,x1 - 6\,x2 + 10\,x3 - 8\,x4 - 8\,x5 = 61\}$

$$eqns := \{-3\,x3 + x4 + 6\,x5 = -10,\ -x1 + 2\,x2 - x3 + 2\,x4 + 5\,x5 = -1, 2\,x1 - 4\,x2 + 3\,x3 - 4\,x4 - 11\,x5 = 28, 3\,x1 - 6\,x2 + 10\,x3 - 8\,x4 - 8\,x5 = 61\} \tag{32}$$

> $GenerateMatrix(eqns, [x1, x2, x3, x4, x5])$

$$\begin{bmatrix} 0 & 0 & -3 & 1 & 6 \\ -1 & 2 & -1 & 2 & 5 \\ 2 & -4 & 3 & -4 & -11 \\ 3 & -6 & 10 & -8 & -8 \end{bmatrix}, \begin{bmatrix} -10 \\ -1 \\ 28 \\ 61 \end{bmatrix} \tag{33}$$

> $LinearSolve(\%)$

$$\begin{bmatrix} \frac{549}{5} + 2\,\_t4_2 \\ \_t4_2 \\ \frac{133}{5} \\ \frac{331}{5} \\ \frac{3}{5} \end{bmatrix} \tag{34}$$

> $eqns2 := \{2x + 2y + z = 1,\ x + y + 3z = 2, x - y + 2z = 3\}$

$$eqns2 := \{x - y + 2z = 3, x + y + 3z = 2, 2x + 2y + z = 1\} \tag{35}$$

> $GenerateMatrix(eqns2, [x, y, z])$

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 3 \\ 2 & 2 & 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \tag{36}$$

> $LinearSolve(\%)$

$$\begin{bmatrix} 1 \\ -\frac{4}{5} \\ \frac{3}{5} \end{bmatrix} \tag{37}$$

> $GaussianElimination\left(\begin{bmatrix} 0 & 0 & -3 & 1 & 6 \\ -1 & 2 & -1 & 2 & 5 \\ 2 & -4 & 3 & -4 & -11 \\ 3 & -6 & 10 & -8 & -8 \end{bmatrix}\right)$

$$\begin{bmatrix} -1 & 2 & -1 & 2 & 5 \\ 0 & 0 & -3 & 1 & 6 \\ 0 & 0 & 0 & \frac{1}{3} & 1 \\ 0 & 0 & 0 & 0 & 20 \end{bmatrix} \tag{38}$$

> $ReducedRowEchelonForm\left(\begin{bmatrix} 0 & 0 & -3 & 1 & 6 \\ -1 & 2 & -1 & 2 & 5 \\ 2 & -4 & 3 & -4 & -11 \\ 3 & -6 & 10 & -8 & -8 \end{bmatrix}\right)$

$$\begin{bmatrix} 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (39)$$

> $eq1 := \{x + y - z = 1, 2x - y + 7z = 8, -x + y - 5z = -5\}$

$$eq1 := \{-x + y - 5z = -5, x + y - z = 1, 2x - y + 7z = 8\} \quad (40)$$

> $GenerateMatrix(eq1, [x, y, z])$

$$\begin{bmatrix} -1 & 1 & -5 \\ 1 & 1 & -1 \\ 2 & -1 & 7 \end{bmatrix}, \begin{bmatrix} -5 \\ 1 \\ 8 \end{bmatrix} \quad (41)$$

> $LinearSolve\left(\begin{bmatrix} -1 & 1 & -5 \\ 1 & 1 & -1 \\ 2 & -1 & 7 \end{bmatrix}, \begin{bmatrix} -5 \\ 1 \\ 8 \end{bmatrix}\right); ReducedRowEchelonForm\left(\begin{bmatrix} -1 & 1 & -5 \\ 1 & 1 & -1 \\ 2 & -1 & 7 \end{bmatrix}\right)$

$$\begin{bmatrix} 3 - 2\,\_t6_3 \\ -2 + 3\,\_t6_3 \\ \_t6_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix} \quad (42)$$

> $Determinant\left(\begin{bmatrix} -1 & 1 & -5 \\ 1 & 1 & -1 \\ 2 & -1 & 7 \end{bmatrix}\right)$

$$0 \quad (43)$$

> $eq2 := \{x - y - z = 1, x + z = 2, y + 2z = 3\}$

$$eq2 := \{x + z = 2, y + 2z = 3, x - y - z = 1\} \quad (44)$$

> $GenerateMatrix(eq2, [x, y, z])$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & -1 & -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \quad (45)$$

> $LinearSolve\left(\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & -1 & -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}\right); ReducedRowEchelonForm\left(\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & -1 & -1 \end{bmatrix}\right)$

 # On a une erreur car le système n'admet pas de solution

Error, (in LinearAlgebra:-LinearSolve) inconsistent system

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \tag{46}$$

> $eq3 := \{x - y + z = 4, 4x - 2y + 3z = 2\}$

$$eq3 := \{x - y + z = 4, 4x - 2y + 3z = 2\} \tag{47}$$

> $GenerateMatrix(eq3, [x, y, z])$

$$\begin{bmatrix} 1 & -1 & 1 \\ 4 & -2 & 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \end{bmatrix} \tag{48}$$

> $LinearSolve\left(\begin{bmatrix} 1 & -1 & 1 \\ 4 & -2 & 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \end{bmatrix}\right); ReducedRowEchelonForm\left(\begin{bmatrix} 1 & -1 & 1 \\ 4 & -2 & 3 \end{bmatrix}\right)$

$$\begin{bmatrix} -10 - \_t8_2 \\ \_t8_2 \\ 14 + 2\,\_t8_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \end{bmatrix} \tag{49}$$

> $eq4 := \{x1 + x3 + x4 - 2x5 = 1, 2x1 + x2 + 3x3 - x4 + x5 = 0, 3x1 - x2 + 4x3 + x4 + x5 = 1\}$

$$eq4 := \{x1 + x3 + x4 - 2x5 = 1, 2x1 + x2 + 3x3 - x4 + x5 = 0, 3x1 - x2 + 4x3 + x4 + x5 = 1\} \tag{50}$$

> $GenerateMatrix(eq4, [x1, x2, x3, x4, x5])$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & -2 \\ 2 & 1 & 3 & -1 & 1 \\ 3 & -1 & 4 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \tag{51}$$

> $LinearSolve\left(\begin{bmatrix} 1 & 0 & 1 & 1 & -2 \\ 2 & 1 & 3 & -1 & 1 \\ 3 & -1 & 4 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right);$

$$ReducedRowEchelonForm\left(\begin{bmatrix} 1 & 0 & 1 & 1 & -2 \\ 2 & 1 & 3 & -1 & 1 \\ 3 & -1 & 4 & 1 & 1 \end{bmatrix}\right)$$

$$\begin{bmatrix} 3 - \frac{7\,\_t9_4}{2} + 8\,\_t9_5 \\ \frac{\_t9_4}{2} + \_t9_5 \\ -2 + \frac{5\,\_t9_4}{2} - 6\,\_t9_5 \\ \_t9_4 \\ \_t9_5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{7}{2} & -8 \\ 0 & 1 & 0 & -\frac{1}{2} & -1 \\ 0 & 0 & 1 & -\frac{5}{2} & 6 \end{bmatrix} \quad \textbf{(52)}$$

```
> restart; with(LinearAlgebra)
```

[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, **(53)**
BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column,
ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix,
CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy,
CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal,
DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues,
Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm,
FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic,
GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix,
HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix,
IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary,
JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition,
LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential,
MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower,
MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply,
NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot,
PopovForm, ProjectionMatrix, QRDecomposition, RandomMatrix, RandomVector, Rank,
RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation,
RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues,
SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis,
SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm,

UnitVector, *VandermondeMatrix*, *VectorAdd*, *VectorAngle*, *VectorMatrixMultiply*, *VectorNorm*, *VectorScalarMultiply*, *ZeroMatrix*, *ZeroVector*, *Zip*]

> $p := a \cdot x^3 + b \cdot x^2 + c \cdot x + d$

$$p := a\,x^3 + b\,x^2 + c\,x + d \tag{54}$$

> $seq(subs(x = i, p), i = 1..4)$

$$a + b + c + d,\ 8\,a + 4\,b + 2\,c + d,\ 27\,a + 9\,b + 3\,c + d,\ 64\,a + 16\,b + 4\,c + d \tag{55}$$

> $eq := \{a + b + c + d = 2, 8\,a + 4\,b + 2\,c + d = 3, 27\,a + 9\,b + 3\,c + d = -1, 64\,a + 16\,b + 4\,c + d = 1\}$

$$eq := \{a + b + c + d = 2, 8\,a + 4\,b + 2\,c + d = 3, 27\,a + 9\,b + 3\,c + d = -1, 64\,a + 16\,b + 4\,c + d = 1\} \tag{56}$$

> $GenerateMatrix(eq, [a, b, c, d])$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \\ 64 & 16 & 4 & 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ -1 \\ 1 \end{bmatrix} \tag{57}$$

> $LinearSolve\left(\begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \\ 64 & 16 & 4 & 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ -1 \\ 1 \end{bmatrix}\right)$

$$\begin{bmatrix} \frac{11}{6} \\ -\frac{27}{2} \\ \frac{86}{3} \\ -15 \end{bmatrix} \tag{58}$$

> $DotProduct\left(\begin{bmatrix} \frac{11}{6} \\ -\frac{27}{2} \\ \frac{86}{3} \\ -15 \end{bmatrix}, Vector([seq(x^{(4-i-1)}, i = 0..3)])\right)$

$$-15 + \frac{11}{6}\,x^3 - \frac{27}{2}\,x^2 + \frac{86}{3}\,x \tag{59}$$

> $N := 10; a := -5; b := 5$

$$N := 10$$

$$a := -5$$
$$b := 5 \tag{60}$$

> $E := \left[seq\left(a + \frac{i\cdot(b-a)}{N-1}, i = 0..N-1\right)\right]; F := Vector\left(\left[seq\left(\frac{1}{1+E[i]^2}, i = 1..N\right)\right]\right)$

$$E := \left[-5, -\frac{35}{9}, -\frac{25}{9}, -\frac{5}{3}, -\frac{5}{9}, \frac{5}{9}, \frac{5}{3}, \frac{25}{9}, \frac{35}{9}, 5\right]$$

$$F := \begin{bmatrix} \frac{1}{26} \\ \frac{81}{1306} \\ \frac{81}{706} \\ \frac{9}{34} \\ \frac{81}{106} \\ \frac{81}{106} \\ \frac{9}{34} \\ \frac{81}{706} \\ \frac{81}{1306} \\ \frac{1}{26} \end{bmatrix} \tag{61}$$

> $M := VandermondeMatrix(E)$

$$M := \left[\left[1, -5, 25, -125, 625, -3125, 15625, -78125, 390625, -1953125\right],\right.$$
$$\left[1, -\frac{35}{9}, \frac{1225}{81}, -\frac{42875}{729}, \frac{1500625}{6561}, -\frac{52521875}{59049}, \frac{1838265625}{531441}, -\frac{64339296875}{4782969},\right.$$
$$\left.\frac{2251875390625}{43046721}, -\frac{78815638671875}{387420489}\right],$$
$$\left[1, -\frac{25}{9}, \frac{625}{81}, -\frac{15625}{729}, \frac{390625}{6561}, -\frac{9765625}{59049}, \frac{244140625}{531441}, -\frac{6103515625}{4782969},\right.$$
$$\left.\frac{152587890625}{43046721}, -\frac{3814697265625}{387420489}\right], \tag{62}$$

$$\left[1, -\frac{5}{3}, \frac{25}{9}, -\frac{125}{27}, \frac{625}{81}, -\frac{3125}{243}, \frac{15625}{729}, -\frac{78125}{2187}, \frac{390625}{6561}, -\frac{1953125}{19683}\right],$$

$$\left[1, -\frac{5}{9}, \frac{25}{81}, -\frac{125}{729}, \frac{625}{6561}, -\frac{3125}{59049}, \frac{15625}{531441}, -\frac{78125}{4782969}, \frac{390625}{43046721}, -\frac{1953125}{387420489}\right],$$

$$\left[1, \frac{5}{9}, \frac{25}{81}, \frac{125}{729}, \frac{625}{6561}, \frac{3125}{59049}, \frac{15625}{531441}, \frac{78125}{4782969}, \frac{390625}{43046721}, \frac{1953125}{387420489}\right],$$

$$\left[1, \frac{5}{3}, \frac{25}{9}, \frac{125}{27}, \frac{625}{81}, \frac{3125}{243}, \frac{15625}{729}, \frac{78125}{2187}, \frac{390625}{6561}, \frac{1953125}{19683}\right],$$

$$\left[1, \frac{25}{9}, \frac{625}{81}, \frac{15625}{729}, \frac{390625}{6561}, \frac{9765625}{59049}, \frac{244140625}{531441}, \frac{6103515625}{4782969}, \frac{152587890625}{43046721}, \frac{3814697265625}{387420489}\right],$$

$$\left[1, \frac{35}{9}, \frac{1225}{81}, \frac{42875}{729}, \frac{1500625}{6561}, \frac{52521875}{59049}, \frac{1838265625}{531441}, \frac{64339296875}{4782969}, \frac{2251875390625}{43046721}, \frac{78815638671875}{387420489}\right],$$

$$\left[1, 5, 25, 125, 625, 3125, 15625, 78125, 390625, 1953125\right]\Big]$$

> $v := LinearSolve(M, F); X := Vector\left(\left[seq\left(x^i, i = 0..N-1\right)\right]\right)$

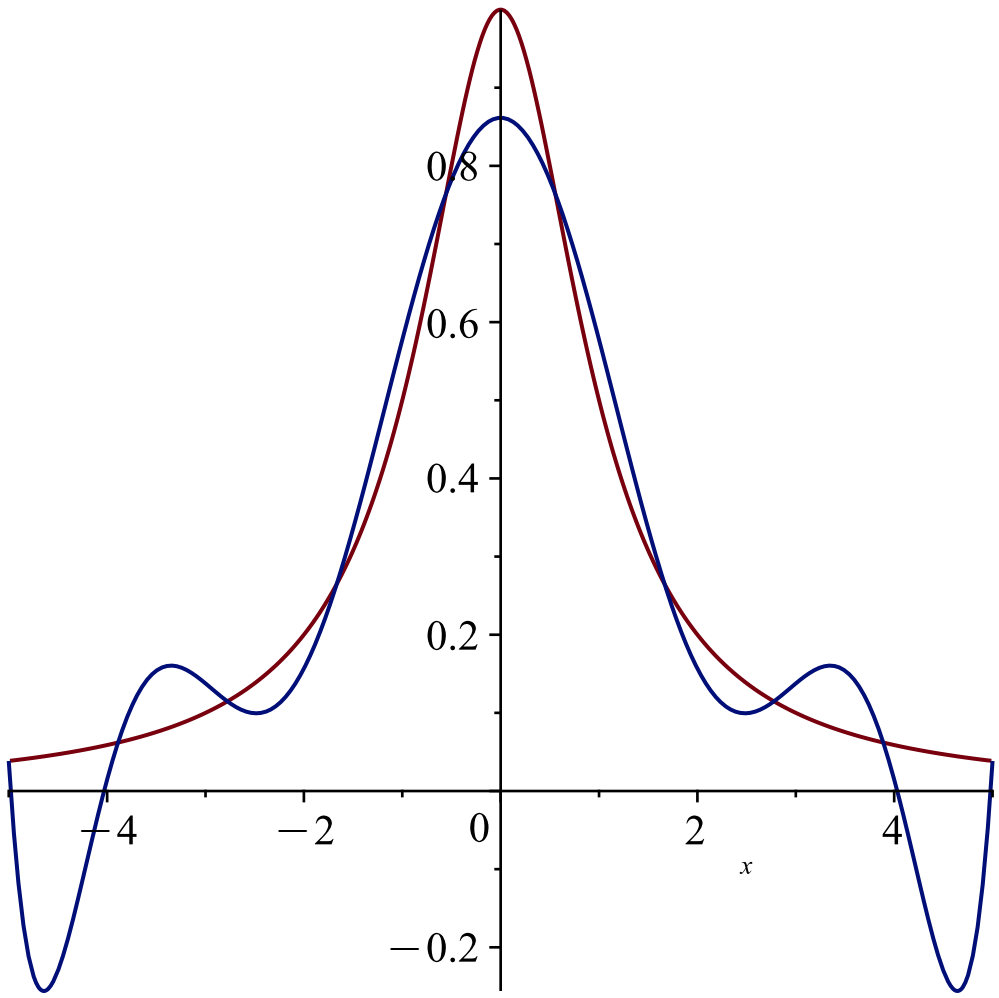
$$v := \begin{bmatrix} \frac{74435570719}{86398461344} \\ 0 \\ -\frac{14274621297}{43199230672} \\ 0 \\ \frac{530979543}{10799807668} \\ 0 \\ -\frac{124180047}{43199230672} \\ 0 \\ \frac{4782969}{86398461344} \\ 0 \end{bmatrix}$$

$$X := \begin{bmatrix} 1 \\ x \\ x^2 \\ x^3 \\ x^4 \\ x^5 \\ x^6 \\ x^7 \\ x^8 \\ x^9 \end{bmatrix} \tag{63}$$

> $DotProduct(v, X)$

$$\frac{74435570719}{86398461344} - \frac{14274621297}{43199230672}\,x^2 + \frac{530979543}{10799807668}\,x^4 - \frac{124180047}{43199230672}\,x^6 + \frac{4782969}{86398461344}\,x^8 \tag{64}$$

> $plot\left(\left[\frac{1}{1+x^2}, \frac{74435570719}{86398461344} - \frac{14274621297}{43199230672}\,x^2 + \frac{530979543}{10799807668}\,x^4 - \frac{124180047}{43199230672}\,x^6 + \frac{4782969}{86398461344}\,x^8\right], x = -5..5\right)$

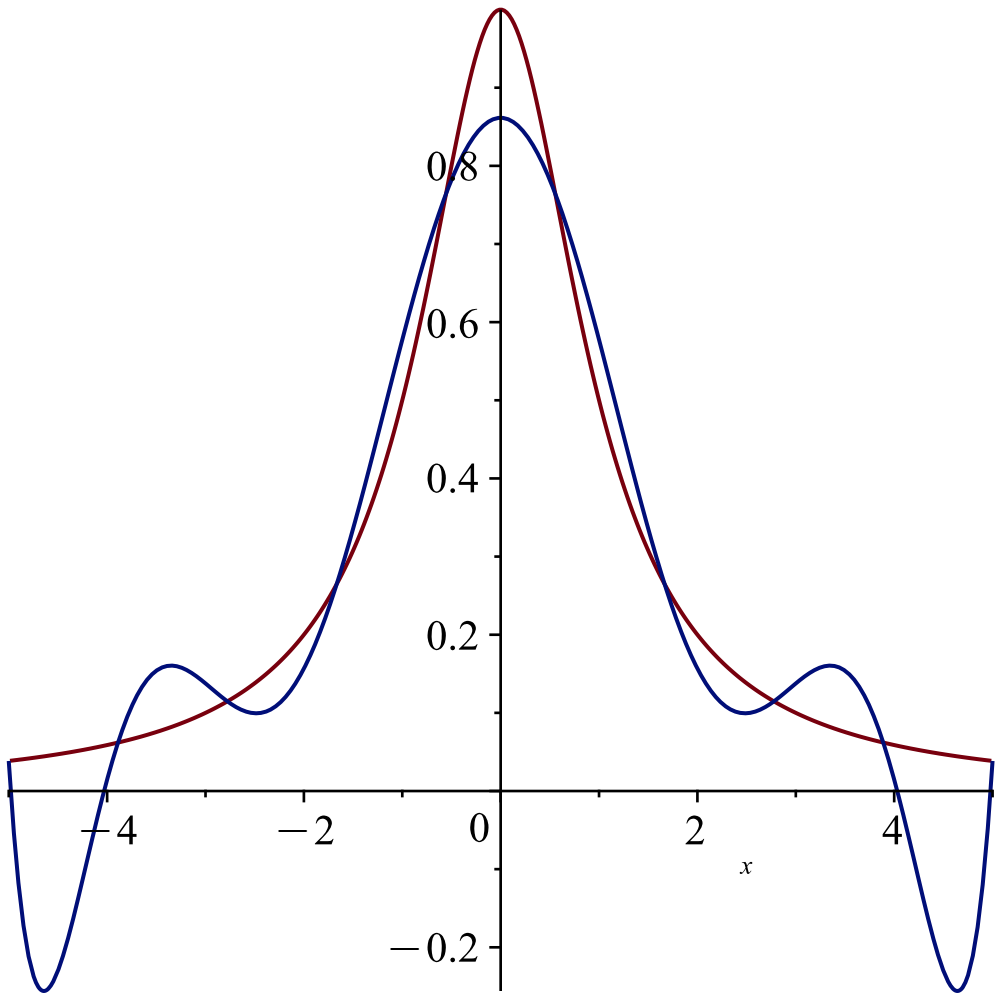



```
> interpo := proc(f, a, b, N)
  local E, F, i, M, v;
  E := [seq(a + i·(b − a)/(N − 1), i = 0 ..N − 1)];
  F := Vector([seq(f(E[i]), i = 1 ..N)]);
  M := VandermondeMatrix(E);
  v := LinearSolve(M, F);
  DotProduct(v, Vector([seq(x^(i − 1), i = 1 ..N)]))
  end proc:
```

```
> interpo(x→1/(1 + x^2), −5, 5, 10)
```

$$\frac{74435570719}{86398461344} - \frac{14274621297}{43199230672}\,x^2 + \frac{530979543}{10799807668}\,x^4 - \frac{124180047}{43199230672}\,x^6 + \frac{4782969}{86398461344}\,x^8 \tag{65}$$

```
> plot([1/(1 + x^2), 74435570719/86398461344 − 14274621297/43199230672 x^2
  + 530979543/10799807668 x^4 − 124180047/43199230672 x^6
  + 4782969/86398461344 x^8], x = −5 ..5)
```

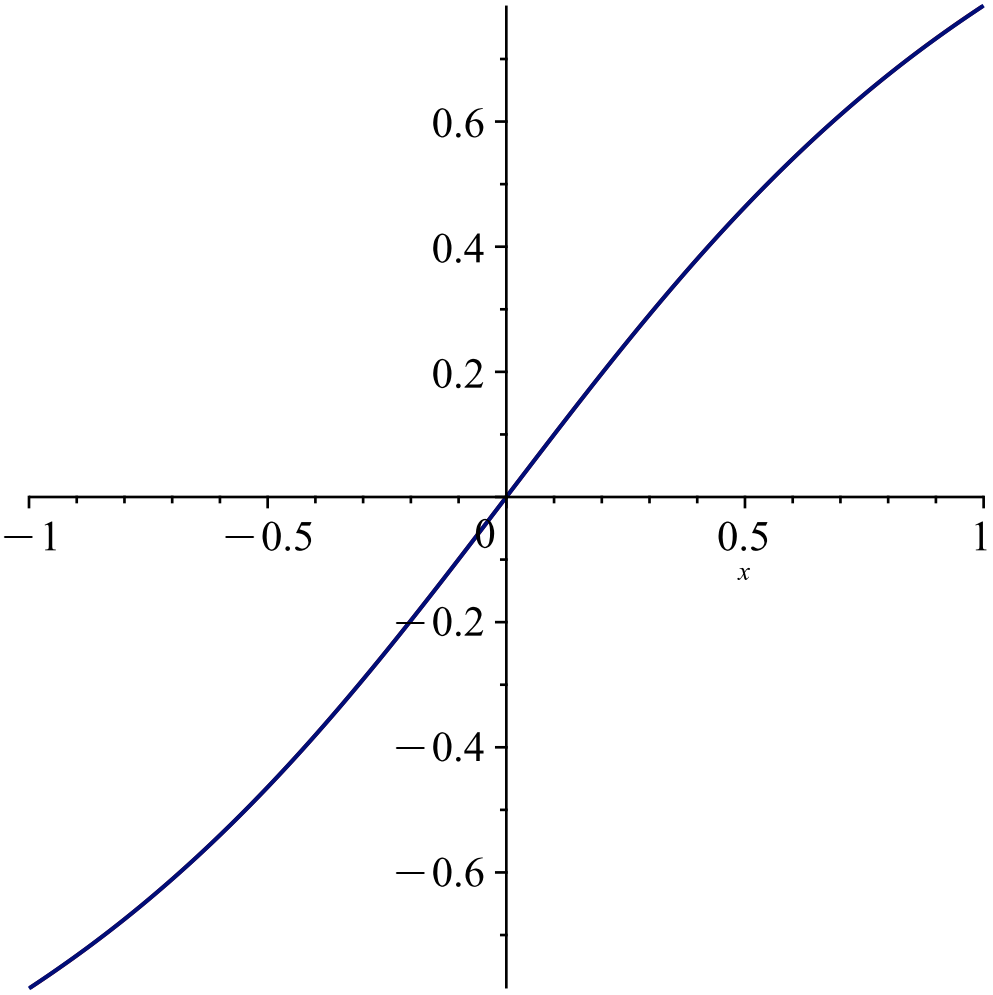


> $p := interpo(x \rightarrow \arctan(x), -1, 1, 10)$

$$p := \left(\frac{35\,\pi}{131072} - \frac{6615 \arctan\left(\frac{1}{3}\right)}{8192} + \frac{178605 \arctan\left(\frac{1}{9}\right)}{16384} + \frac{5103 \arctan\left(\frac{5}{9}\right)}{40960} - \frac{3645 \arctan\left(\frac{7}{9}\right)}{229376} \right) x + \left(-\frac{29061\,\pi}{1146880} + \frac{718497 \arctan\left(\frac{1}{3}\right)}{10240} - \frac{3324969 \arctan\left(\frac{1}{9}\right)}{20480} - \frac{165483 \arctan\left(\frac{5}{9}\right)}{14336} + \frac{429381 \arctan\left(\frac{7}{9}\right)}{286720} \right) x^3 + \left(\frac{102789\,\pi}{327680} - \frac{8063469 \arctan\left(\frac{1}{3}\right)}{20480} + \frac{28258227 \arctan\left(\frac{1}{9}\right)}{40960} + \frac{2473497 \arctan\left(\frac{5}{9}\right)}{20480} - \frac{1449981 \arctan\left(\frac{7}{9}\right)}{81920} \right) x^5 + \left(-\frac{177147\,\pi}{163840} \right. \tag{66}$$

$$+\frac{6908733\arctan\left(\frac{1}{3}\right)}{10240}-\frac{21789081\arctan\left(\frac{1}{9}\right)}{20480}-\frac{531441\arctan\left(\frac{5}{9}\right)}{2048}$$
$$+\frac{15411789\arctan\left(\frac{7}{9}\right)}{286720}\Bigg)x^7+\Bigg(\frac{4782969\,\pi}{4587520}-\frac{14348907\arctan\left(\frac{1}{3}\right)}{40960}$$
$$+\frac{43046721\arctan\left(\frac{1}{9}\right)}{81920}+\frac{43046721\arctan\left(\frac{5}{9}\right)}{286720}-\frac{43046721\arctan\left(\frac{7}{9}\right)}{1146880}\Bigg)x^9$$

> $plot([\arctan(x), p], x=-1..1)$

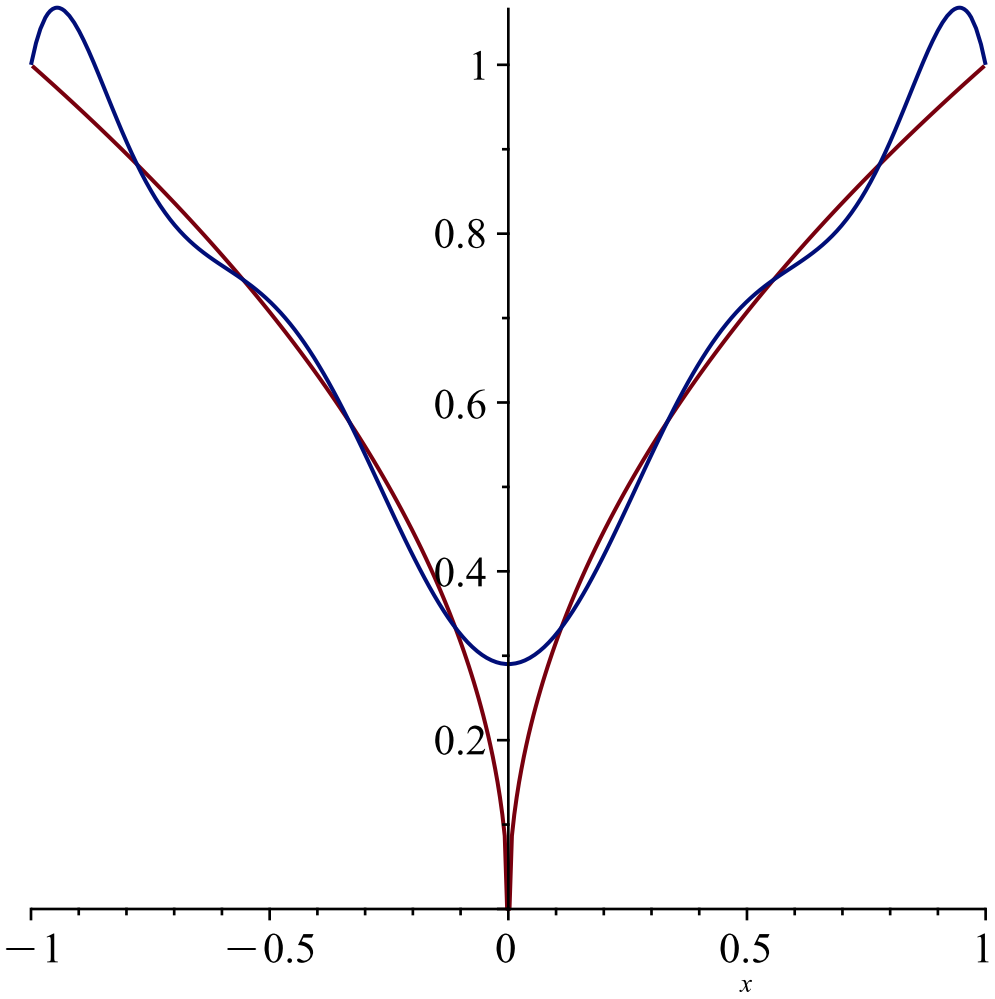


> $q := interpo(x\to\mathrm{sqrt}(\mathrm{abs}(x)), -1, 1, 10)$

$$q := \frac{13265}{32768}-\frac{735\sqrt{3}}{8192}-\frac{135\sqrt{7}}{32768}+\frac{189\sqrt{5}}{8192}+\Bigg(-\frac{1753119}{286720}+\frac{79833\sqrt{3}}{10240}$$
$$+\frac{15903\sqrt{7}}{40960}-\frac{30645\sqrt{5}}{14336}\Bigg)x^2+\Bigg(\frac{2195991}{81920}-\frac{895941\sqrt{3}}{20480}-\frac{375921\sqrt{7}}{81920}$$
$$+\frac{91611\sqrt{5}}{4096}\Bigg)x^4+\Bigg(-\frac{1791153}{40960}+\frac{767637\sqrt{3}}{10240}+\frac{570807\sqrt{7}}{40960}-\frac{98415\sqrt{5}}{2048}\Bigg)x^6 \quad (67)$$

$$+\left(\frac{27103491}{1146880}-\frac{1594323\sqrt{3}}{40960}-\frac{1594323\sqrt{7}}{163840}+\frac{1594323\sqrt{5}}{57344}\right)x^{8}$$

```
> plot( [sqrt(abs(x)), q], x =-1 ..1 )
```




```
>
```